CHAPTER 1

Appendix A: Graphs in Economics

A glance through the pages of this book should convince you that there are a lot of graphs in economics. The language of graphs is one means of presenting economic ideas. If you are already familiar with graphs, you will have no difficulty with this aspect of your study. If you have never used graphs or have not used them in some time, this appendix will help you feel comfortable with the graphs you will encounter in this text.

1. HOW TO CONSTRUCT AND INTERPRET GRAPHS

LEARNING OBJECTIVES

1. Understand how graphs show the relationship between two or more variables and explain how a graph elucidates the nature of the relationship.
2. Define the slope of a curve.
3. Distinguish between a movement along a curve, a shift in a curve, and a rotation in a curve.

Much of the analysis in economics deals with relationships between variables. A variable is simply a quantity whose value can change. A graph is a pictorial representation of the relationship between two or more variables. The key to understanding graphs is knowing the rules that apply to their construction and interpretation. This section defines those rules and explains how to draw a graph.

1.1 Drawing a Graph

To see how a graph is constructed from numerical data, we will consider a hypothetical example. Suppose a college campus has a ski club that organizes day-long bus trips to a ski area about 100 miles from the campus. The club leases the bus and charges $10 per passenger for a round trip to the ski area. In addition to the revenue the club collects from passengers, it also receives a grant of $200 from the school’s student government for each day the bus trip is available. The club thus would receive $200 even if no passengers wanted to ride on a particular day.

The table in Figure 1.1 shows the relationship between two variables: the number of students who ride the bus on a particular day and the revenue the club receives from a trip. In the table, each combination is assigned a letter (A, B, etc.); we will use these letters when we transfer the information from the table to a graph.
**FIGURE 1.1 Ski Club Revenues**

The ski club receives $10 from each passenger riding its bus for a trip to and from the ski area plus a payment of $200 from the student government for each day the bus is available for these trips. The club’s revenues from any single day thus equal $200 plus $10 times the number of passengers. The table relates various combinations of the number of passengers and club revenues.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Number of passengers</th>
<th>Club revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>$200</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>300</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
<td>500</td>
</tr>
<tr>
<td>E</td>
<td>40</td>
<td>600</td>
</tr>
</tbody>
</table>

We can illustrate the relationship shown in the table with a graph. The procedure for showing the relationship between two variables, like the ones in Figure 1.1, on a graph is illustrated in Figure 1.2. Let us look at the steps involved.

**FIGURE 1.2 Plotting a Graph**

Here we see how to show the information given in Figure 1.1 in a graph.

**Step 1. Draw and Label the Axes**

The two variables shown in the table are the number of passengers taking the bus on a particular day and the club’s revenue from that trip. We begin our graph in Panel (a) of Figure 1.2 by drawing two axes to form a right angle. Each axis will represent a variable. The axes should be carefully labeled to reflect what is being measured on each axis.

It is customary to place the independent variable on the horizontal axis and the dependent variable on the vertical axis. Recall that, when two variables are related, the dependent variable is the one that changes in response to changes in the independent variable. Passengers generate revenue, so we can consider the number of passengers as the independent variable and the club’s revenue as the dependent variable. The number of passengers thus goes on the horizontal axis; the club’s revenue from a trip goes...
on the vertical axis. In some cases, the variables in a graph cannot be considered independent or dependent. In those cases, the variables may be placed on either axis; we will encounter such a case in the chapter that introduces the production possibilities model. In other cases, economists simply ignore the rule; we will encounter that case in the chapter that introduces the model of demand and supply. The rule that the independent variable goes on the horizontal axis and the dependent variable goes on the vertical usually holds, but not always.

The point at which the axes intersect is called the origin of the graph. Notice that in Figure 1.2 the origin has a value of zero for each variable.

In drawing a graph showing numeric values, we also need to put numbers on the axes. For the axes in Panel (a), we have chosen numbers that correspond to the values in the table. The number of passengers ranges up to 40 for a trip; club revenues from a trip range from $200 (the payment the club receives from student government) to $600. We have extended the vertical axis to $800 to allow some changes we will consider below. We have chosen intervals of 10 passengers on the horizontal axis and $100 on the vertical axis. The choice of particular intervals is mainly a matter of convenience in drawing and reading the graph; we have chosen the ones here because they correspond to the intervals given in the table.

We have drawn vertical lines from each of the values on the horizontal axis and horizontal lines from each of the values on the vertical axis. These lines, called gridlines, will help us in Step 2.

Step 2. Plot the Points

Each of the rows in the table in Figure 1.1 gives a combination of the number of passengers on the bus and club revenue from a particular trip. We can plot these values in our graph.

We begin with the first row, A, corresponding to zero passengers and club revenue of $200, the payment from student government. We read up from zero passengers on the horizontal axis to $200 on the vertical axis and mark point A. This point shows that zero passengers result in club revenues of $200.

The second combination, B, tells us that if 10 passengers ride the bus, the club receives $300 in revenue from the trip—$100 from the $10-per-passenger charge plus the $200 from student government. We start at 10 passengers on the horizontal axis and follow the gridline up. When we travel up in a graph, we are traveling with respect to values on the vertical axis. We travel up by $300 and mark point B.

Points in a graph have a special significance. They relate the values of the variables on the two axes to each other. Reading to the left from point B, we see that it shows $300 in club revenue. Reading down from point B, we see that it shows 10 passengers. Those values are, of course, the values given for combination B in the table.

We repeat this process to obtain points C, D, and E. Check to be sure that you see that each point corresponds to the values of the two variables given in the corresponding row of the table.

The graph in Panel (b) is called a scatter diagram. A scatter diagram shows individual points relating values of the variable on one axis to values of the variable on the other.

Step 3. Draw the Curve

The final step is to draw the curve that shows the relationship between the number of passengers who ride the bus and the club’s revenues from the trip. The term “curve” is used for any line in a graph that shows a relationship between two variables.

We draw a line that passes through points A through E. Our curve shows club revenues; we shall call it $R_1$. Notice that $R_1$ is an upward-sloping straight line. Notice also that $R_1$ intersects the vertical axis at $200 (point A). The point at which a curve intersects an axis is called the intercept of the curve. We often refer to the vertical or horizontal intercept of a curve; such intercepts can play a special role in economic analysis. The vertical intercept in this case shows the revenue the club would receive on a day it offered the trip and no one rode the bus.

To check your understanding of these steps, we recommend that you try plotting the points and drawing $R_1$ for yourself in Panel (a). Better yet, draw the axes for yourself on a sheet of graph paper and plot the curve.

1.2 The Slope of a Curve

In this section, we will see how to compute the slope of a curve. The slopes of curves tell an important story: they show the rate at which one variable changes with respect to another.
The **slope** of a curve equals the ratio of the change in the value of the variable on the vertical axis to the change in the value of the variable on the horizontal axis, measured between two points on the curve. You may have heard this called “the rise over the run.” In equation form, we can write the definition of the slope as

EQUATION 1.1

\[
\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}}
\]

Equation 1.1 is the first equation in this text. Figure 1.3 provides a short review of working with equations. The material in this text relies much more heavily on graphs than on equations, but we will use equations from time to time. It is important that you understand how to use them.

**FIGURE 1.3 Reading and Using Equations**

Many equations in economics begin in the form of Equation 1.1, with the statement that one thing (in this case the slope) equals another (the vertical change divided by the horizontal change). In this example, the equation is written in words. Sometimes we use symbols in place of words. The basic idea though, is always the same: the term represented on the left side of the equals sign equals the term on the right side. In Equation 1.1 there are three variables: the slope, the vertical change, and the horizontal change. If we know the values of two of the three, we can compute the third. In the computation of slopes that follow, for example, we will use values for the two variables on the right side of the equation to compute the slope.

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Figure 1.4 shows $R_1$ and the computation of its slope between points B and D. Point B corresponds to 10 passengers on the bus; point D corresponds to 30. The change in the horizontal axis when we go from B to D thus equals 20 passengers. Point B corresponds to club revenues of $300; point D corresponds to club revenues of $500. The change in the vertical axis equals $200. The slope thus equals $200/20 passengers, or $10/passenger.
FIGURE 1.4 Computed the Slope of a Curve

1. Select two points; we have selected points B and D.
2. The slope equals the vertical change divided by the horizontal change between the two points.
3. Between points B and D, the slope equals \( \frac{\$200}{20 \text{ passengers}} = \$10/\text{passenger} \).
4. The slope of this curve is the price per passenger. The fact that it is positive suggests a positive relationship between revenue per trip and the number of passengers riding the bus. Because the slope of this curve is \( \$10/\text{passenger} \) between any two points on the curve, the relationship between club revenue per trip and the number of passengers is linear.

The slope of the curve tells us the amount by which revenues rise with an increase in the number of passengers. It should come as no surprise that this amount equals the price per passenger. Adding a passenger adds \$10 to the club’s revenues.

Notice that we can compute the slope of \( R_1 \) between any two points on the curve and get the same value; the slope is constant. Consider, for example, points A and E. The vertical change between these points is \$400 \ (we go from revenues of \$200 \ at A to revenues of \$600 \ at E). The horizontal change is 40 passengers (from zero passengers at A to 40 at E). The slope between A and E thus equals \$400/(40 \text{ passengers}) = \$10/\text{passenger} \). We get the same slope regardless of which pair of points we pick on \( R_1 \) to compute the slope. The slope of \( R_1 \) can be considered a constant, which suggests that it is a straight line. When the curve showing the relationship between two variables has a constant slope, we say there is a linear relationship between the variables. A linear curve is a curve with constant slope.

The fact that the slope of our curve equals \$10/\text{passenger} \ tells us something else about the curve—\$10/\text{passenger} \ is a positive, not a negative, value. A curve whose slope is positive is upward sloping. As we travel up and to the right along \( R_1 \), we travel in the direction of increasing values for both variables. A positive relationship between two variables is one in which both variables move in the same direction. Positive relationships are sometimes called direct relationships. There is a positive relationship between club revenues and passengers on the bus. We will look at a graph showing a negative relationship between two variables in the next section.

1.3 A Graph Showing a Negative Relationship

A negative relationship is one in which two variables move in opposite directions. A negative relationship is sometimes called an inverse relationship. The slope of a curve describing a negative relationship is always negative. A curve with a negative slope is always downward sloping.

As an example of a graph of a negative relationship, let us look at the impact of the cancellation of games by the National Basketball Association during the 1998–1999 labor dispute on the earnings of one player: Shaquille O’Neal. During the 1998–1999 season, O’Neal was the center for the Los Angeles Lakers.
O’Neal’s salary with the Lakers in 1998–1999 would have been about $17,220,000 had the 82 scheduled games of the regular season been played. But a contract dispute between owners and players resulted in the cancellation of 32 games. Mr. O’Neal’s salary worked out to roughly $210,000 per game, so the labor dispute cost him well over $6 million. Presumably, he was able to eke out a living on his lower income, but the cancellation of games cost him a great deal.

We show the relationship between the number of games canceled and O’Neal’s 1998–1999 basketball earnings graphically in Figure 1.5. Canceling games reduced his earnings, so the number of games canceled is the independent variable and goes on the horizontal axis. O’Neal’s earnings are the dependent variable and go on the vertical axis. The graph assumes that his earnings would have been $17,220,000 had no games been canceled (point A, the vertical intercept). Assuming that his earnings fell by $210,000 per game canceled, his earnings for the season were reduced to $10,500,000 by the cancellation of 32 games (point B). We can draw a line between these two points to show the relationship between games canceled and O’Neal’s 1998–1999 earnings from basketball. In this graph, we have inserted a break in the vertical axis near the origin. This allows us to expand the scale of the axis over the range from $10,000,000 to $18,000,000. It also prevents a large blank space between the origin and an income of $10,500,000—there are no values below this amount.

**FIGURE 1.5 CANCELING GAMES AND REDUCING SHAQUILLE O’NEAL’S EARNINGS**

If no games had been canceled during the 1998–1999 basketball season, Shaquille O’Neal would have earned $17,220,000 (point A). Assuming that his salary for the season fell by $210,000 for each game canceled, the cancellation of 32 games during the dispute between NBA players and owners reduced O’Neal’s earnings to $10,500,000 (point B).

What is the slope of the curve in Figure 1.5? We have data for two points, A and B. At A, O’Neal’s basketball salary would have been $17,220,000. At B, it is $10,500,000. The vertical change between points A and B equals -$6,720,000. The change in the horizontal axis is from zero games canceled at A to 32 games canceled at B. The slope is thus

\[
\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-6,720,000}{32 \text{ games}} = -\frac{6,720,000}{32} = -210,000 \, \text{per game}
\]
Notice that this time the slope is negative, hence the downward-sloping curve. As we travel down and to the right along the curve, the number of games canceled rises and O'Neal’s salary falls. In this case, the slope tells us the rate at which O’Neal lost income as games were canceled.

The slope of O’Neal’s salary curve is also constant. That means there was a linear relationship between games canceled and his 1998–1999 basketball earnings.

1.4 Shifting a Curve

When we draw a graph showing the relationship between two variables, we make an important assumption. We assume that all other variables that might affect the relationship between the variables in our graph are unchanged. When one of those other variables changes, the relationship changes, and the curve showing that relationship shifts.

Consider, for example, the ski club that sponsors bus trips to the ski area. The graph we drew in Figure 1.2 shows the relationship between club revenues from a particular trip and the number of passengers on that trip, assuming that all other variables that might affect club revenues are unchanged. Let us change one. Suppose the school’s student government increases the payment it makes to the club to $400 for each day the trip is available. The payment was $200 when we drew the original graph. Panel (a) of Figure 1.6 shows how the increase in the payment affects the table we had in Figure 1.1; Panel (b) shows how the curve shifts. Each of the new observations in the table has been labeled with a prime: A’, B’, etc. The curve R₁ shifts upward by $200 as a result of the increased payment. A shift in a curve implies new values of one variable at each value of the other variable. The new curve is labeled R₂. With 10 passengers, for example, the club’s revenue was $300 at point B on R₁. With the increased payment from the student government, its revenue with 10 passengers rises to $500 at point B’ on R₂. We have a shift in the curve.

**FIGURE 1.6 Shifting a Curve: An Increase in Revenues**

The table in Panel (a) shows the new level of revenues the ski club receives with varying numbers of passengers as a result of the increased payment from student government. The new curve is shown in dark purple in Panel (b). The old curve is shown in light purple.
It is important to distinguish between shifts in curves and movements along curves. A **movement along a curve** is a change from one point on the curve to another that occurs when the dependent variable changes in response to a change in the independent variable. If, for example, the student government is paying the club $400 each day it makes the ski bus available and 20 passengers ride the bus, the club is operating at point \( C' \) on \( R_2 \). If the number of passengers increases to 30, the club will be at point \( D' \) on the curve. This is a movement along a curve; the curve itself does not shift.

Now suppose that, instead of increasing its payment, the student government eliminates its payments to the ski club for bus trips. The club’s only revenue now comes from the $10 it charges to each passenger. We have again changed one of the variables we were holding unchanged, so we get another shift in our revenue curve. The table in Panel (a) of Figure 1.7 shows how the reduction in the student government’s payments affects club revenues. The new values are shown as combinations \( A'' \) through \( E'' \) on the new curve, \( R_3 \), in Panel (b). Once again we have a shift in a curve, this time from \( R_1 \) to \( R_3 \).

**FIGURE 1.7 Shifting a Curve: A Reduction in Revenues**

The table in Panel (a) shows the impact on ski club revenues of an elimination of support from the student government for ski bus trips. The club’s only revenue now comes from the $10 it charges to each passenger. The new combinations are shown as \( A'' \) – \( E'' \). In Panel (b) we see that the original curve relating club revenue to the number of passengers has shifted down.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Number of passengers</th>
<th>Club revenue (with no payment from student government)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A'' )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( B'' )</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>( C'' )</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>( D'' )</td>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>( E'' )</td>
<td>40</td>
<td>400</td>
</tr>
</tbody>
</table>

The shifts in Figure 1.6 and Figure 1.7 left the slopes of the revenue curves unchanged. That is because the slope in all these cases equals the price per ticket, and the ticket price remains unchanged. Next, we shall see how the slope of a curve changes when we rotate it about a single point.

### 1.5 Rotating a Curve

A **rotation of a curve** occurs when we change its slope, with one point on the curve fixed. Suppose, for example, the ski club changes the price of its bus rides to the ski area to $30 per trip, and the payment from the student government remains $200 for each day the trip is available. This means the club’s revenues will remain $200 if it has no passengers on a particular trip. Revenue will, however, be different when the club has passengers. Because the slope of our revenue curve equals the price per ticket, the slope of the revenue curve changes.

Panel (a) of Figure 1.8 shows what happens to the original revenue curve, \( R_1 \), when the price per ticket is raised. Point \( A \) does not change; the club’s revenue with zero passengers is unchanged. But with 10 passengers, the club’s revenue would rise from $300 (point \( B \) on \( R_1 \)) to $500 (point \( B' \) on \( R_4 \)). With 20 passengers, the club’s revenue will now equal $800 (point \( C' \) on \( R_4 \)).
A curve is said to rotate when a single point remains fixed while other points on the curve move; a rotation always changes the slope of a curve. Here an increase in the price per passenger to $30 would rotate the revenue curve from $R_1$ to $R_4$ in Panel (a). The slope of $R_4$ is $30$ per passenger.

The new revenue curve $R_4$ is steeper than the original curve. Panel (b) shows the computation of the slope of the new curve between points $B'$ and $C'$. The slope increases to $30$ per passenger—the new price of a ticket. The greater the slope of a positively sloped curve, the steeper it will be.

We have now seen how to draw a graph of a curve, how to compute its slope, and how to shift and rotate a curve. We have examined both positive and negative relationships. Our work so far has been with linear relationships. Next we will turn to nonlinear ones.

**KEY TAKEAWAYS**

- A graph shows a relationship between two or more variables.
- An upward-sloping curve suggests a positive relationship between two variables. A downward-sloping curve suggests a negative relationship between two variables.
- The slope of a curve is the ratio of the vertical change to the horizontal change between two points on the curve. A curve whose slope is constant suggests a linear relationship between two variables.
- A change from one point on the curve to another produces a movement along the curve in the graph. A shift in the curve implies new values of one variable at each value of the other variable. A rotation in the curve implies that one point remains fixed while the slope of the curve changes.
The following table shows the relationship between the number of gallons of gasoline people in a community are willing and able to buy per week and the price per gallon. Plot these points in the grid provided and label each point with the letter associated with the combination. Notice that there are breaks in both the vertical and horizontal axes of the grid. Draw a line through the points you have plotted. Does your graph suggest a positive or a negative relationship? What is the slope between A and B? Between B and C? Between A and C? Is the relationship linear?

<table>
<thead>
<tr>
<th>Combination</th>
<th>Price per gallon</th>
<th>Number of gallons (per week)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1.00</td>
<td>1,000</td>
</tr>
<tr>
<td>B</td>
<td>$1.20</td>
<td>900</td>
</tr>
<tr>
<td>C</td>
<td>$1.40</td>
<td>800</td>
</tr>
</tbody>
</table>

Now suppose you are given the following information about the relationship between price per gallon and the number of gallons per week gas stations in the community are willing to sell.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Price per gallon</th>
<th>Number of gallons (per week)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>$1.00</td>
<td>800</td>
</tr>
<tr>
<td>E</td>
<td>$1.20</td>
<td>900</td>
</tr>
<tr>
<td>F</td>
<td>$1.40</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Plot these points in the grid provided and draw a curve through the points you have drawn. Does your graph suggest a positive or a negative relationship? What is the slope between D and E? Between E and F? Between D and F? Is this relationship linear?
ANSWER TO TRY IT!

Here is the first graph. The curve’s downward slope tells us there is a negative relationship between price and the quantity of gasoline people are willing and able to buy. This curve, by the way, is a demand curve (the next one is a supply curve). We will study demand and supply soon; you will be using these curves a great deal. The slope between A and B is \(-0.002\) \((\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = -\frac{0.20}{100})\). The slope between B and C and between A and C is the same. That tells us the curve is linear, which, of course, we can see—it is a straight line.

Here is the supply curve. Its upward slope tells us there is a positive relationship between price per gallon and the number of gallons per week gas stations are willing to sell. The slope between D and E is 0.002 \((\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = 0.20/100)\). Because the curve is linear, the slope is the same between any two points, for example, between E and F and between D and F.

2. NONLINEAR RELATIONSHIPS AND GRAPHS
WITHOUT NUMBERS

LEARNING OBJECTIVES

1. Understand nonlinear relationships and how they are illustrated with nonlinear curves.
2. Explain how to estimate the slope at any point on a nonlinear curve.
3. Explain how graphs without numbers can be used to understand the nature of relationships between two variables.

In this section we will extend our analysis of graphs in two ways: first, we will explore the nature of nonlinear relationships; then we will have a look at graphs drawn without numbers.

2.1 Graphs of Nonlinear Relationships

In the graphs we have examined so far, adding a unit to the independent variable on the horizontal axis always has the same effect on the dependent variable on the vertical axis. When we add a passenger riding the ski bus, the ski club’s revenues always rise by the price of a ticket. The cancellation of one more game in the 1998–1999 basketball season would always reduce Shaquille O’Neal’s earnings by $210,000. The slopes of the curves describing the relationships we have been discussing were constant; the relationships were linear.
Many relationships in economics are nonlinear. A **nonlinear relationship** between two variables is one for which the slope of the curve showing the relationship changes as the value of one of the variables changes. A **nonlinear curve** is a curve whose slope changes as the value of one of the variables changes.

Consider an example. Suppose Felicia Alvarez, the owner of a bakery, has recorded the relationship between her firm’s daily output of bread and the number of bakers she employs. The relationship she has recorded is given in the table in Panel (a) of Figure 1.9. The corresponding points are plotted in Panel (b). Clearly, we cannot draw a straight line through these points. Instead, we shall have to draw a nonlinear curve like the one shown in Panel (c).
FIGURE 1.9 A Nonlinear Curve

The table in Panel (a) shows the relationship between the number of bakers Felicia Alvarez employs per day and the number of loaves of bread produced per day. This information is plotted in Panel (b). This is a nonlinear relationship; the curve connecting these points in Panel (c) (Loaves of bread produced) has a changing slope.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Bakers per day</th>
<th>Loaves of bread produced per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>700</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>900</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>1,000</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>1,050</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>1,075</td>
</tr>
</tbody>
</table>

Inspecting the curve for loaves of bread produced, we see that it is upward sloping, suggesting a positive relationship between the number of bakers and the output of bread. But we also see that the curve becomes flatter as we travel up and to the right along it; it is nonlinear and describes a nonlinear relationship.
How can we estimate the slope of a nonlinear curve? After all, the slope of such a curve changes as we travel along it. We can deal with this problem in two ways. One is to consider two points on the curve and to compute the slope between those two points. Another is to compute the slope of the curve at a single point.

When we compute the slope of a curve between two points, we are really computing the slope of a straight line drawn between those two points. In Figure 1.10, we have computed slopes between pairs of points A and B, C and D, and E and F on our curve for loaves of bread produced. These slopes equal 400 loaves/baker, 200 loaves/baker, and 50 loaves/baker, respectively. They are the slopes of the dashed-line segments shown. These dashed segments lie close to the curve, but they clearly are not on the curve. After all, the dashed segments are straight lines. Our curve relating the number of bakers to daily bread production is not a straight line; the relationship between the bakery’s daily output of bread and the number of bakers is nonlinear.

**FIGURE 1.10  Estimating Slopes for a Nonlinear Curve**

We can estimate the slope of a nonlinear curve between two points. Here, slopes are computed between points A and B, C and D, and E and F. When we compute the slope of a nonlinear curve between two points, we are computing the slope of a straight line between those two points. Here the lines whose slopes are computed are the dashed lines between the pairs of points.

Every point on a nonlinear curve has a different slope. To get a precise measure of the slope of such a curve, we need to consider its slope at a single point. To do that, we draw a line tangent to the curve at that point. A **tangent line** is a straight line that touches, but does not intersect, a nonlinear curve at only one point. The slope of a tangent line equals the slope of the curve at the point at which the tangent line touches the curve.

Consider point D in Panel (a) of Figure 1.11. We have drawn a tangent line that just touches the curve showing bread production at this point. It passes through points labeled M and N. The vertical change between these points equals 300 loaves of bread; the horizontal change equals two bakers. The slope of the tangent line equals 150 loaves of bread/baker (300 loaves/2 bakers). The slope of our bread production curve at point D equals the slope of the line tangent to the curve at this point. In Panel (b), we have sketched lines tangent to the curve for loaves of bread produced at points B, D, and F. Notice that these tangent lines get successively flatter, suggesting again that the slope of the curve is falling as we travel up and to the right along it.
FIGURE 1.11  Tangent Lines and the Slopes of Nonlinear Curves

Because the slope of a nonlinear curve is different at every point on the curve, the precise way to compute slope is to draw a tangent line; the slope of the tangent line equals the slope of the curve at the point the tangent line touches the curve. In Panel (a), the slope of the tangent line is computed for us: it equals 150 loaves/baker. Generally, we will not have the information to compute slopes of tangent lines. We will use them as in Panel (b), to observe what happens to the slope of a nonlinear curve as we travel along it. We see here that the slope falls (the tangent lines become flatter) as the number of bakers rises.

Notice that we have not been given the information we need to compute the slopes of the tangent lines that touch the curve for loaves of bread produced at points B and F. In this text, we will not have occasion to compute the slopes of tangent lines. Either they will be given or we will use them as we did here—to see what is happening to the slopes of nonlinear curves.

In the case of our curve for loaves of bread produced, the fact that the slope of the curve falls as we increase the number of bakers suggests a phenomenon that plays a central role in both microeconomic and macroeconomic analysis. As we add workers (in this case bakers), output (in this case loaves of bread) rises, but by smaller and smaller amounts. Another way to describe the relationship between the number of workers and the quantity of bread produced is to say that as the number of workers increases, the output increases at a decreasing rate. In Panel (b) of Figure 1.11 we express this idea with a graph, and we can gain this understanding by looking at the tangent lines, even though we do not have specific numbers. Indeed, much of our work with graphs will not require numbers at all.

We turn next to look at how we can use graphs to express ideas even when we do not have specific numbers.

2.2  Graphs Without Numbers

We know that a positive relationship between two variables can be shown with an upward-sloping curve in a graph. A negative or inverse relationship can be shown with a downward-sloping curve. Some relationships are linear and some are nonlinear. We illustrate a linear relationship with a curve whose slope is constant; a nonlinear relationship is illustrated with a curve whose slope changes. Using these basic ideas, we can illustrate hypotheses graphically even in cases in which we do not have numbers with which to locate specific points.

Consider first a hypothesis suggested by recent medical research: eating more fruits and vegetables each day increases life expectancy. We can show this idea graphically. Daily fruit and vegetable consumption (measured, say, in grams per day) is the independent variable; life expectancy (measured in years) is the dependent variable. Panel (a) of Figure 1.12 shows the hypothesis, which suggests a positive relationship between the two variables. Notice the vertical intercept on the curve we have drawn; it implies that even people who eat no fruit or vegetables can expect to live at least a while!
Figure 1.12  Graphs Without Numbers

We often use graphs without numbers to suggest the nature of relationships between variables. The graphs in the four panels correspond to the relationships described in the text.

Panel (b) illustrates another hypothesis we hear often: smoking cigarettes reduces life expectancy. Here the number of cigarettes smoked per day is the independent variable; life expectancy is the dependent variable. The hypothesis suggests a negative relationship. Hence, we have a downward-sloping curve.

Now consider a general form of the hypothesis suggested by the example of Felicia Alvarez’s bakery: increasing employment each period increases output each period, but by smaller and smaller amounts. As we saw in Figure 1.9, this hypothesis suggests a positive, nonlinear relationship. We have drawn a curve in Panel (c) of Figure 1.12 that looks very much like the curve for bread production in Figure 1.11. It is upward sloping, and its slope diminishes as employment rises.

Finally, consider a refined version of our smoking hypothesis. Suppose we assert that smoking cigarettes does reduce life expectancy and that increasing the number of cigarettes smoked per day reduces life expectancy by a larger and larger amount. Panel (d) shows this case. Again, our life expectancy curve slopes downward. But now it suggests that smoking only a few cigarettes per day reduces life expectancy only a little but that life expectancy falls by more and more as the number of cigarettes smoked per day increases.

We have sketched lines tangent to the curve in Panel (d). The slopes of these tangent lines are negative, suggesting the negative relationship between smoking and life expectancy. They also get steeper as the number of cigarettes smoked per day rises. Whether a curve is linear or nonlinear, a steeper curve is one for which the absolute value of the slope rises as the value of the variable on the horizontal axis rises. When we speak of the absolute value of a negative number such as −4, we ignore the minus
sign and simply say that the absolute value is 4. The absolute value of −8, for example, is greater than the absolute value of −4, and a curve with a slope of −8 is steeper than a curve whose slope is −4.

Thus far our work has focused on graphs that show a relationship between variables. We turn finally to an examination of graphs and charts that show values of one or more variables, either over a period of time or at a single point in time.

**KEY TAKEAWAYS**

- The slope of a nonlinear curve changes as the value of one of the variables in the relationship shown by the curve changes.
- A nonlinear curve may show a positive or a negative relationship.
- The slope of a curve showing a nonlinear relationship may be estimated by computing the slope between two points on the curve. The slope at any point on such a curve equals the slope of a line drawn tangent to the curve at that point.
- We can illustrate hypotheses about the relationship between two variables graphically, even if we are not given numbers for the relationships. We need only draw and label the axes and then draw a curve consistent with the hypothesis.

**TRY IT!**

Consider the following curve drawn to show the relationship between two variables, A and B (we will be using a curve like this one in the next chapter). Explain whether the relationship between the two variables is positive or negative, linear or nonlinear. Sketch two lines tangent to the curve at different points on the curve, and explain what is happening to the slope of the curve.
The relationship between variable A shown on the vertical axis and variable B shown on the horizontal axis is negative. This is sometimes referred to as an inverse relationship. Variables that give a straight line with a constant slope are said to have a linear relationship. In this case, however, the relationship is nonlinear. The slope changes all along the curve. In this case the slope becomes steeper as we move downward to the right along the curve, as shown by the two tangent lines that have been drawn. As the quantity of B increases, the quantity of A decreases at an increasing rate.